

The New Weibull Handbook

**Reliability & Statistical Analysis for Predicting Life, Safety,
Survivability, Risk, Cost and Warranty Claims**

**Fourth Edition by Dr. Robert B. Abernethy
Author and Publisher**

The New Weibull Handbook, Fourth Edition

Copyright © 1993, 1996, 1998, 2000 by Robert B. Abernethy

All rights reserved. Reproduction or translation of any part of this work beyond that permitted by Section 107 or 108 of the 1976 United States Copyright Act without permission of the copyright author is unlawful. Requests for permission should be addressed to the author.

Library of Congress Cataloging in Publication Data

96-202898 TA169.A35 1996 620/.0045 21
Abernethy, Robert B. The New Weibull Handbook, 2nd ed.
1v. (various pagings) :ill.; 28 cm
Includes bibliographical references and index.
ISBN 0-9653062-1-6
1. Reliability (Engineering). 2. Statistical Methods
3. Weibull distribution
Book source: ad

ISBN 0-9653062-1-6

Published and distributed by Robert B. Abernethy

536 Oyster Road, North Palm Beach, Florida 33408-4328
Phone/Fax: 1-561-842-4082, E-mail: weibull@worldnet.att.net

Also distributed by:

1. SAE International (Society of Automotive Engineers) Phone: 724-772-8597, Fax: 724-776-4955, E-mail: profdev@sae.org and <http://www.sae.org>
2. Wes Fulton, Phone/Fax: 310-548-6358, Web page: <http://www.weibullnews.com>
3. Paul Barringer, Phone: 281-852-6810, E-mail: hpaul@barringer1.com and <http://www.barringer1.com>
4. ASME, Phone: 800-THE-ASME
5. Carl Tarum, Phone: 517-791-4405, E-mail: cdtarum@juno.com
6. Reliability Analysis Center, Phone: 888-RAC-USER (722-8737)
7. Williams Enterprises, Phone: 916-885-5322

Copyrights and Trademarks: Windows is trademarked by Microsoft, Inc. YBATH is trademarked by Carl Tarum. The New Weibull Handbook copyright belongs to Dr. Robert B. Abernethy. Copyrights for SuperSMITH software, WinSMITH Weibull, WinSMITH Visual, MonteCarloSMITH and "Playtime with SuperSMITH" belong to Fulton Findings.

Printing History

First Edition, December 1993, Printings to June 1996

Second Edition, July 1996, 9 Printings to January 1998

Third Edition, September 1998, 13 Printings May 2000

Fourth Edition, September 2000, Printings November 2000, February 2001, April, September, January 2002, March, July, August.

Dedication:

This book is dedicated to my wife, Sally, for her encouragement and patience. Further, it is dedicated to all my friends and colleagues that helped so much. Finally, it is dedicated to the reader with the hope it will have "some utility," to use Professor Weibull's words.

And lastly we have a special note for the Weibull Workshop students for whom we designed this Handbook...

Weibull Workshop Students:

If you are fortunate enough to have obtained or borrowed the Handbook before your Workshop, studying the Handbook before the Workshop is very helpful. **Please** read Chapters 1-8 & 10. We will cover the basic standard method the first day, Chapters 1-4. For the second day, study Chapters 5-7 (7 is least important). For the third day, Chapters 8 & 10. (Most important material for the third day are the Duane-AMSAA and Kaplan-Meier Models.) Chapter 9 is case studies which you should browse. I won't ask you to do the problems in the Handbook. We will do a few of them in class. Please bring a clear plastic ruler, a calculator, and a problem from your work. The data should be in a single column, positive values are failures, negative are suspensions. You may bring the data in ASCII format, or a spreadsheet file (Excel) or WinSMITH Weibull files. The file may be on a diskette or in your laptop.

Weibull Workshop Classwork:

Day One Chapters 1-4

1. Handbook Problems: 2-1, 2-2, 2-3, 3-2...(Do this last one with your neighbor)
2. Fatigue Wire Weibull Experiment. We will analyze this data tomorrow.
3. Read the Handbook through Chapter 7.

Day Two Chapters 5 & 6...& 7...maybe?

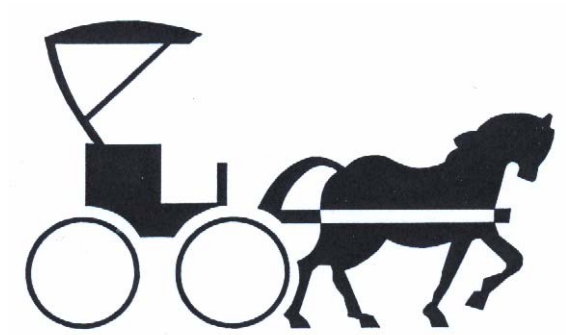
1. Do the "PlayTIME with SuperSMITH" on a computer. Try to complete problems 1 through 14, but do not worry if you do not finish as there will be more Playtime tomorrow.
2. Read the Handbook Chapters 8 & 10. The Duane-AMSAA & Kaplan-Meier models in Chapter 8 are the most important material for tonight. Skim Chapter 9 for interest. Your Ph.D. oral exam is based on Chapter 10.

Day Three Chapters 7, 8 & 10.

1. Ph.D. Oral Examination
2. Finish "PlayTIME" tutorial. You may select the problems. I recommend 1-14, Problem 2-Continued, 15-18, 21, 24, & (26... if you work on warranty claims.) Your own data and problems have priority.

"Now in building of chaises, I tell you what,
There is always *somewhere* a weakest spot,-
In hub, tire, felloe, in spring or thill,
In panel, or crossbar, or floor, or sill,
In screw, bolt, throughbrace,-lurking still,
Find it somewhere you must and will,-
Above or below, or within or without,-
And that's the reason, beyond a doubt,
A chaise *breaks down*, but doesn't *wear out*.

Oliver Wendell Holmes... "The One-Hoss Shay"



PREFACE

Background Almost two decades have passed since the *USAF Weibull Analysis Handbook* was published. In the years that followed, the author frequently taught Weibull analysis using the USAF Handbook as the text, encouraged by both the United States Air Force and Pratt & Whitney. In 1987, the author retired early to devote full time to research and lecturing in Weibull Analysis. It has been most enjoyable and interesting. The best part has been meeting outstanding engineers from many industries with new and different applications for Weibull methods. The students inspire additional research and methods development. This in turn required rewriting and updating the USAF Handbook producing *The New Weibull Handbook*. This document is the fourth edition. As always, it contains the latest methods, techniques, and applications that have been developed by the author, Wes Fulton, Paul Barringer, Bob Rock, Wayne Nelson, Carl Tarum, M.A. Vasan, Geoff Cole and David Navell and many others.

The New Weibull Handbook has become the national and international standard for Weibull analysis. It is in daily use throughout the world. Thousands of copies have been distributed. Users include military, automotive, medical, electronics, materials & composites research, aerospace, electrical power, nuclear power, dental research, advertising, bearings, compressors, v-belts, chain drives and on and on. Many organizations have adopted the Handbook and software implementing its methods as standard including the Federal Aviation Administration, Harley Davidson, Motorola, TVA, Honeywell, BICC General, GKN Automotive, Dupont, Meritor Automotive, Teleflex, Guidant Cardiac Pacemaker (CPI), Daimler Chrysler, General Electric, Rolls Royce, Gates Rubber, the US Naval Air Arm (NAVAIR), the US Air Force at SAALC and WPAFB, and Dow Chemical. The author with the Society of Automotive Engineering, (SAE) has provided many free copies of the Handbook to universities in Europe, North America, and Australia. The author hereby extends this offer to university libraries worldwide. Wes Fulton provides free demonstration copies of the SuperSMITH software on our web site.

The **SuperSMITH** software is the most significant technical breakthrough. Wes Fulton has created friendly, easy-to-use software that does almost everything. *The New Weibull Handbook* provides an understanding and reference for SuperSMITH. The computer software has made application so much easier. It has also made teaching Weibull easier, more efficient, and more fun. Within the Weibull workshops, many more applications and case studies can be treated without the drudgery of hand calculations and plotting. As this document is the workbook for the Weibull Workshops, many references to the software capabilities are included. This referencing may appear to be inappropriate, but the reliance on software in the workshops requires it. If the student using *The New Weibull Handbook* desires, he may avoid almost all hand calculations and hand plotting by using SuperSMITH. The new "*Playtime with SuperSMITH*" computer tutorial enhances the Handbook and provides many step-by-step case studies illustrating all the methods. Alternatively, the reader may use this Handbook to write "home-grown" software. Outputs are easily validated with benchmark case studies found in both demonstration and full versions of the SuperSMITH software.

The **Fourth Edition** contains many new methods and examples. Wayne Nelson contributed his latest technology, graphical repair analysis, as Appendix M. Bob Rock of PACCAR developed an extremely useful life data analysis logic diagram which takes you step-by-step to the best methods and analysis for your particular data. You will find the logic diagram in Chapter 10. In Chapter 3 there is a new section on detecting outliers, wild points in a data set. Thanks to Paul Barringer's research we now have goodness of fit for the Crow-AMSAA model and have changed our best practice on Crow-AMSAA from MLE to regression. Paul also contributed his latest application, production process control using Weibull analysis. Extreme value statistics are included in Chapter 8. The marriage of Wes Fulton's small sample compensation for the likelihood ratio test and the likelihood contour test with the author's MLE-Reduced Bias Adjustment is completed. This allows likelihood analysis to be used with very small samples with extreme accuracy for the first time. Predictive intervals for failure and warranty forecasts have been improved. The New Weibull Handbook is unique in recommending best practices wherever there are alternative methods available. For example median rank regression is still recommended as the engineering standard, even with the small sample bias eliminated from MLE, as it is simple and provides a good plot of the data. However, the exceptions where maximum likelihood estimates are better are indicated. Pivotal confidence intervals, suggested by Dick Rudy of Daimler Chrysler, are now best practice for small samples. Warranty claims forecasting includes seasonal usage and adjustment for systems aging out of warranty. With complete warranty data, Wayne Nelson's Graphical Repair Analysis method should be considered

as described in Appendix M. Cost analysis is described for production process reliability, failure forecasts, Crow-AMSAA, and optimal parts replacement. Some of the methods in the earlier editions are now obsolete and have been omitted. To make the Handbook more readable, all the heavy mathematics will be found in the appendices. Asterisks on section subtitles indicate advanced topics. The novice reader may skip these sections. As the workbook for the Weibull Analysis Workshops, the format includes much white space for notes and calculations. Dozens of case studies are employed to present the methodology.

Fourth Edition 2002 printings: New technology has been developed to (1) allow total uncertainty intervals estimates and (2) to determine the “p” value of r squared for the most sensitive goodness of fit and distribution analysis. A biography Waloddi Weibull is included. Further, Paul Barringer has taken the most important reference texts and papers from my library and put them on his Website for downloading. <http://www.barringer1.com> They are indicated by a \$ sign.

Thank You: Many friends, students, experts, and clients have helped and contributed much to the Handbook. For me, all this support has been overwhelming. Everyone helped! Thank you all. Some are mentioned here:

- First, my associate, Wes Fulton, not only for his friendly software, but also for his research and development of new Weibull methods.
- Second, our associate, Paul Barringer, for his help, comments, reviews, research, and contributions.
- The second printing of the fourth edition is completely reformatted with many thanks to Pam Kinstle, an editor at the Reliability Analysis Center. We are indebted to Pamela the significant improvements.
- Many contributed sections of the Handbook: Bob Rock, PACCAR; David B. Smith, Detroit Edison; Bob Scanlon, Southern Pacific (now with the US Postal Service); Sten-Åke Irell, Volvo Flygmotor; Marie Stanton, FPL; David Weber, Consultant; Carl Tarum, Delphia; Hans Iwand, Union Pacific; Drs. Memis and Lautenschlager, Dentists at Northwestern University; Richard Rudy, Chrysler; Kenneth Young, HydriL; Jim Keener and Mel Thomas, FPL; Bill Pritchard, TVA; Shashank Kolhatkar, Borg Warner; Charlie Williams, Florida Power; Geoffrey Cole and David Navell, Rolls Royce; David Williamson, Dow Chemical; M.A. Vasan, Meritor Automotive; David B. Smith, Detroit Edison; and David Langanke, Honeywell Engines.
- Some served as reviewers and provided encouragement: Dr. Stephen F. Duffy, Cleveland State University; Charles T. Sica, Torrington Bearing; Richard Rudy, Daimler-Chrysler; Charles H. Medlin, Consultant; Jim Dockendorff and David Saunders, General Electric; Casey Brace, Puget Sound Power & Light; Mary Rowsee, Chrysler; Romma Hartman, Honda; Joe Dzekevich, 3COM; and Jim Hollingsworth, my neighbor. The Society of Automotive Engineers professional development staff provided encouragement. Wright Patterson Air Force Base has supported Weibull developments all the way back to Professor Weibull; Jim Day and Bob Henderson from WPAFB have encouraged the author for many years.
- This edition is improved by incorporating the suggestions from the excellent constructive review of the Handbook by Dr. Alan Winterbottom for the Royal Statistical Society Journal A (1997), 160, Part 2. The author is indebted.
- Particular thanks go to Dr. Wayne Nelson, Dr. Jerald Lawless, and Dr. Larry Crow, world famous statisticians, for their excellent support and also for being the Keynote Speakers at the last three SAE Weibull User's Conferences.
- Most of what I know about statistics I learned long ago from my professor at Imperial College, London, Dr. George Bernard. Recently, he has been most helpful with the application of the likelihood function. (August 6, 2002, received an E-mail from Professor Dennis Farlie that Dr. Barnard just passed on.)
- I also thank my wife, Sally, for being very patient and my daughter, Nancy, for her proof reading.
- None of the above are in any way responsible for errors in the Handbook. The author takes all the blame.

Please call or write with questions, comments, and constructive criticism about *The New Weibull Handbook*. I would like to hear from you:

Dr. Robert B. Abernethy

536 Oyster Road

North Palm Beach, Florida 33408-4328

Fax & Answerphone: 561-842-4082 E-mail: weibull@worldnet.att.net

Table of Contents

CHAPTER 1 AN OVERVIEW OF WEIBULL ANALYSIS	1-1
1.1 OBJECTIVE	1-1
1.2 BACKGROUND	1-1
1.3 EXAMPLES	1-2
1.4 SCOPE	1-2
1.5 ADVANTAGES OF WEIBULL ANALYSIS	1-3
1.6 LIFE DATA AND AGING: TIME OR CYCLES OR MILEAGE	1-3
1.7 FAILURE DISTRIBUTION	1-4
1.8 FAILURE FORECASTS AND PREDICTIONS	1-5
1.9 ENGINEERING CHANGE TEST SUBSTANTIATION	1-6
1.10 MAINTENANCE PLANNING	1-6
1.11 SYSTEM ANALYSIS AND MATH MODELS	1-6
1.12 WEIBULLS WITH CURVED DATA	1-7
1.13 WEIBULLS WITH CORNERS AND DOGLEGS	1-8
1.14 WEIBAYES	1-8
1.15 SMALL SAMPLE WEIBULLS	1-9
1.16 UPDATING WEIBULLS	1-9
1.17 DEFICIENT (DIRTY) DATA	1-9
1.18 ESTABLISHING THE WEIBULL LINE, CHOOSING THE FIT METHOD	1-9
1.19 RELATED METHODS AND PROBLEMS	1-9
1.20 SUMMARY	1-10
CHAPTER 2 PLOTTING THE DATA AND INTERPRETING THE PLOT	2-1
2.1 FOREWORD	2-1
2.2 WEIBULL DATA	2-1
2.3 THE WEIBULL PLOTS SCALES	2-2
2.4 η (ETA) AND β (BETA)	2-2
2.5 WEIBULL ANALYSIS - AN EXAMPLE	2-4
2.6 MEDIAN RANKS	2-4
2.7 THE WEIBULL PLOT	2-5
2.8 "B" LIFE	2-6
2.9 SUSPENDED TEST ITEMS	2-6
2.10 BERNARD'S APPROXIMATION	2-7
2.11 SUSPENSIONS INCREASE ETA	2-7
2.12 INTERPRETING THE WEIBULL PLOT	2-8
2.13 $\beta < 1$ IMPLIES INFANT MORTALITY	2-9
2.14 $\beta = 1.0$ IMPLIES RANDOM FAILURES	2-9
2.15 $1.0 < \beta < 4.0$ IMPLIES EARLY WEAR OUT	2-10
2.16 $\beta > 4.0$ IMPLIES OLD AGE (RAPID) WEAR OUT	2-10
2.17 UNKNOWN WEIBULL MODES MAY BE "COVERED"	2-11
2.18 WEIBULL PAPER AND ITS CONSTRUCTION	2-11
2.19 WEIBULL ANALYSIS - THE STANDARD METHOD	2-13
2.20 PROBLEMS	2-13

CHAPTER 3 DIRTY DATA, "BAD" WEIBULLS, AND UNCERTAINTIES	3-1
3.1 FOREWORD	3-1
3.2 SMALL SAMPLE UNCERTAINTIES	3-1
3.2.1 Goodness of Fit	3-3
3.3 SUSPENSIONS	3-5
3.4 SUSPECT OUTLIERS	3-5
3.5 CURVED WEIBULLS AND THE CORRECTION	3-6
3.6 CURVED WEIBULLS AND THE LOG NORMAL DISTRIBUTION	3-10
3.7 DATA INCONSISTENCIES AND MULTIMODE FAILURES	3-13
3.7.1 Low-Time Failures	3-14
3.7.2 Close Serial Numbers	3-15
3.7.3 Mixtures of Failure Modes	3-15
3.8 STEEP SLOPES HIDE PROBLEMS	3-17
3.9 CONCLUSION	3-18
3.10 PROBLEMS	3-18
CHAPTER 4 FAILURE FORECASTING = RISK ANALYSIS	4-1
4.1 SITUATION	4-1
4.2 DEFINITION	4-1
4.3 FORECASTING TECHNIQUES	4-1
4.4 CALCULATING FAILURE FORECASTS	4-1
4.4.1 Expected Failures Now	55
4.4.2 Failure Forecast When Failed Units Are Not Replaced	4-3
4.4.3 Failure Forecasts When Failed Units are Replaced, Production Rates, Usage Rates, Warranty Claims	4-4
4.5 FAILURE FORECAST ANALYSIS-SUMMARY	4-4
4.5.1 Case Study 1: Bearing Cage Fracture	4-5
4.5.2 Case Study 2: Bleed System Failures	4-8
4.6 SYSTEM FAILURE FORECAST WITHOUT SIMULATION*	4-13
4.6.1 Case Study 3: Aircraft In-Flight Engine Shutdowns*	4-13
4.7 SYSTEM FAILURE FORECASTS WITH SIMULATION*	4-16
4.7.1 Case Study 4: System Failure Forecast With Simulation*	4-18
4.8 OPTIMAL (LOWEST COST) AND BLOCK REPLACEMENT INTERVALS*	4-20
4.9 PROBLEMS	4-25
CHAPTER 5 MAXIMUM LIKELIHOOD ESTIMATES & OTHER ALTERNATIVES	5-1
5.1 INTRODUCTION	5-1
5.2 MAXIMUM LIKELIHOOD ESTIMATION (MLE)	5-1
5.3 MEDIAN RANK REGRESSION: X ON Y VERSUS Y ON X	5-3
5.4 SPECIAL METHODS: PLOTTING POSITIONS	5-5
5.5 SPECIAL METHODS: MLE WITH REDUCED BIAS ADJUSTMENT (RBA)	5-6
5.6 SPECIAL METHODS: GOSSETT'S STUDENT'S T	5-9
5.7 THE DAUSER SHIFT - UNKNOWN SUSPENSION TIMES	5-9
5.8 SPECIAL METHODS FOR INSPECTION INTERVAL DATA AND COARSE DATA	5-11
5.8.1 Interval Maximum Likelihood Estimation (MLE)	5-12
5.9 SPECIAL METHODS FOR INSPECTION PROBIT DATA	5-12
5.10 DISTRIBUTION ANALYSIS	5-14

CHAPTER 6 WEIBAYES AND WEIBAYES SUBSTANTIATION TESTING	6-1
6.1 FOREWORD	6-1
6.2 WEIBAYES METHOD	6-2
6.3 WEIBAYES - NO FAILURES	6-2
6.4 WEIBAYES WITH FAILURES	6-3
6.5 UNKNOWN FAILURE TIMES	6-4
6.6 WEIBAYES WORRIES AND CONCERNS	6-4
6.7 WEIBAYES CASE STUDIES	6-4
6.8 SUBSTANTIATION AND RELIABILITY TESTING	6-9
6.9 ZERO-FAILURE TEST PLANS FOR SUBSTANTIATION TESTING	6-10
6.10 ZERO-FAILURE TEST PLANS FOR RELIABILITY TESTING	6-12
6.10.1 Re-expression of Reliability Goal to Determine ?	6-12
6.10.2 Designing Test Plans	
6.11 TOTAL TEST TIME	6-14
6.12 TEST-TO-FAILURE VERSUS WEIBAYES ZERO-FAILURE TESTS	6-14
6.13 ONE OR ZERO FAILURE TEST PLANS	6-17
6.14 SUDDEN DEATH TESTS WITH WEIBULL AND WEIBAYES	6-18
6.15 CASE STUDY: COST VS. UNCERTAINTY TRADES	6-19
6.16 NORMAL AND LOGNORMAL TESTS	6-21
6.17 ACCELERATED TESTING	6-21
6.17.1* ACCELERATED STEP-STRESS TEST DATA ANALYSIS	
6.17.2* ACCELERATED TESTING: A METHOD FOR ESTIMATING TEST ACCELERATION FACTOR WITH NO EXISTING IN-SERVICE FAILURES	
6.18 SYSTEM DETERIORATION	6-25
6.19 WEIBULL LIBRARIES AND LESSONS LEARNED	6-27
6.20 WEIBULL LIBRARIES FOR END USERS	6-27
6.21 PROBLEMS	6-28
CHAPTER 7 INTERVAL ESTIMATES	7-1
7.1 CONFIDENCE CONCEPT	7-1
7.2 OTHER INTERVAL ESTIMATES	7-1
7.3 CONFIDENCE INTERVALS FOR B LIVES AND RELIABILITY	7-2
7.3.1 Beta-Binomial Bounds	7-2
7.3.2 Fisher's Matrix Bounds	7-3
7.3.3 Likelihood Ratio Bounds	7-4
7.3.4 Monte Carlo Bounds	7-5
7.3.5 Pivotal Bounds	7-6
7.3.6 Reliability Assurance Interval	7-7
7.3.7 Standard Normal Distribution Confidence Bounds	7-7
7.3.8 Summary of Confidence Bounds for B Life and Reliability	7-7
7.4 CONFIDENCE INTERVALS FOR ETA AND BETA	7-8
7.5 ARE TWO WEIBULL DATA SETS DIFFERENT OR FROM THE SAME	7-8
7.5.1 Double Confidence Bounds do not Overlap	7-9
7.5.2 Likelihood Ratio Test	7-10
7.5.3 Likelihood Contour Plots	7-11
7.6 Total Uncertainty Interval Estimates	7-13
7.6.1 Total Error	7-13
7.6.2 Four Kinds of Error	7-13
7.6.3 Total Uncertainty	7-14

7.6.4. Uncertainty Analysis for Life Data	7-14
7.6.5 Sensitivity Analysis	7-16
7.6.6 The Pre-Test Declaration	7-16
7.6.7. Uncertainty Levels	7-16
7.6.8 Uncertainty Report	7-16
7.6.9 Examples:	7-17
7.7 PROBLEMS - TRUE OR FALSE?	7-21

CHAPTER 8 RELATED MATH MODELS 8-1

8.1 INTRODUCTION	8-1
8.2 BINOMIAL DISTRIBUTION	8-1
8.3 POISSON DISTRIBUTION	8-5
8.4 BINOMIAL BECOMES POISSON...SOMETIMES	8-9
8.5 THE EXPONENTIAL DISTRIBUTION	8-11
8.6 KAPLAN-MEIER SURVIVAL ESTIMATES	8-12
8.7 THE CROW-AMSAA-DUANE RELIABILITY GROWTH MODEL	8-17
8.7.1 Introduction	8-17
8.7.2 Background and History	8-18
8.7.3 CA Model Graphical Solution	8-18
8.7.4 MLE Solutions for Time and Failure Terminated Data	8-20
8.7.5 MLE Solutions for Interval or Grouped Data	8-22
8.7.6 New Research Results on the Crow-AMSAA	8-25
8.7.7 Author's Comment:	8-27
8.7.8 Repairable Systems and Warranties	8-28
8.8 PROBABILISTIC DESIGN	8-28
8.8.1 Strength-Load and Life-Usage Interactions	8-29
8.8.2 Total Life = Crack Life + Crack-to-Rupture Life	8-30
8.9 PRODUCTION PROCESS RELIABILITY	8-30
8.10 EXTREME VALUE STATISTICS	8-32
8.11 BATCH EFFECTS	8-34
8.12 PROBLEMS	8-34

CHAPTER 9 CASE STUDIES AND NEW APPLICATIONS 9-1

9.1 FOREWORD	9-1
9.2 STRESS CORROSION FAILURE FORECASTING	9-1
9.3 OPTIMAL COMPONENT REPLACEMENT - VOLTAGE REGULATORS	9-3
9.4 LOCOMOTIVE POWER UNITS OVERHAUL LIFE	9-4
9.5 LOW CYCLE FATIGUE CRACKS IN TURBINE DISKS	9-10
9.6 COST EFFECTIVE CALIBRATION INTERVALS	9-12
9.7 MACHINE TOOL AND ROBOTIC ACCURACY	9-13
9.8 SHEAR RAM BLOWOUT PREVENTER TESTS	9-15
9.9 FPL TURBOGENERATOR FAILURE	9-18
9.10 TVA BULL RUN FOSSIL PLANT - CONTROLLER CARDS	9-19
9.11 ARE THE BONDED DRIVES FAILING BECAUSE OF COLD WEATHER, OR JUST PLAIN WORN OUT?	9-21
9.11.1 Introduction	9-21
9.11.2 Background and Analysis	9-22
9.11.3 Results	9-22
9.11.4 Conclusions and Recommendations	9-24

9.12 REPAIRABLE SYSTEMS RELIABILITY GROWTH ASSESSMENT	9-25
9.13 FRONT JOUNCE BUMPERS	9-26
9.14 TRANSFER CASE SEAL	9-27
9.15 DENTAL ACRYLIC ADHESIVE FATIGUE	9-28
9.16 DUANE-CROW-AMSAA RELIABILITY MODELING	9-29
9.17 WEIBULL ANALYSIS OF BOILER TUBE FAILURES	9-32
9.18 GAS TURBINE SEAL FAILURES - A BATCH PROBLEM	9-35
9.19 CONVERTING WARRANTY CLAIMS IN WEEKS TO MILES	9-37

CHAPTER 10 SUMMARY 10-1

10.1 THE BEGINNING OF THE END	10-1
10.2 WHICH METHOD? WHAT KIND OF DATA?	10-1
10.3 LOOKING AT THE PLOT, WHAT DO YOU SEE?	10-3
10.4 WHICH DISTRIBUTION IS BEST?	10-4
10.5 SUBSTANTIATION AND ACCELERATED TESTING	10-6
10.6 CONFIDENCE INTERVALS	10-6
10.7 PRESENTATIONS AND REPORTS	10-6
10.8 LOGIC DIAGRAM - FLOWCHART	10-6
10.9 THE END	10-6
10.10 BEST PRACTICE FLOW CHART	10-7

APPENDIX A: GLOSSARY A-1

APPENDIX B: RANK REGRESSION AND CORRELATION METHOD OF WEIBULL ANALYSIS B-1

B.1 METHOD	B-1
B.2 EXAMPLE AND STEP-BY-STEP PROCEDURE	B-1

APPENDIX C: MAXIMUM LIKELIHOOD ESTIMATION* C-1

C.1 FOREWORD	C-1
C.2 STATISTICS, PROBABILITY AND LIKELIHOOD	C-1
C.3 THE LIKELIHOOD FUNCTION	C-1
C.4 MAXIMIZING THE LIKELIHOOD FUNCTION	C-2
C.5 MAXIMUM LIKELIHOOD EXAMPLE	C-4
C.6 INTERVAL MLE	C-6

APPENDIX D: RANK REGRESSION VERSUS MAXIMUM LIKELIHOOD VERSUS MLE-RBAD-1

D.1 BACKGROUND	D-1
D.2 GRAPHIC DISPLAY	D-1
D.3 B LIFE ESTIMATES	D-1
D.4 RISK ANALYSIS	D-1
D.5 BETA & ETA	D-1
D.6 CONFIDENCE INTERVALS	D-1
D.7 BATCH PROBLEMS	D-1
D.8 CONVERGENCE	D-1
D.9 SINGLE FAILURE	D-2
D.10 MATHEMATICAL RIGOR	D-2
D.11 PRESENTATIONS	D-2

D.12 SUMMARY D-2

APPENDIX E: WEIBAYES ANALYSIS E-1

- E.1 FOREWORD E-1
- E.2 WEIBAYES EQUATION WITH NO FAILURES E-1
- E.3 WEIBAYES WITH FAILURES E-2

APPENDIX F: BATCH FAILURES USING THE AGGREGATED CUMULATED HAZARD FUNCTION F-1

- F.1 BATCH FAILURES ON WEIBULL PLOTS F-1
- F.2 BATCH PROBLEMS WITH THE "PRESENT-RISK" METHOD F-2
- F.3 THE ACH METHOD F-3
- F.4 A CASE STUDY: AERO-ENGINES – (LP TURBINE STRAP FAILURES) F-4
- F.5 CONCLUDING REMARKS F-5

APPENDIX G: WEIBULL AND LOG NORMAL MEAN AND VARIANCE G-1

- G.1 RTH MOMENTS G-1
- G.2 WEIBULL MEAN G-2
- G.3 WEIBULL VARIANCE G-3
- G.4 WEIBULL MODE G-3
- G.5 WEIBULL MEDIAN G-3
- G.6 LOG NORMAL MEAN AND STANDARD DEVIATION G-3
- G.7 LOG NORMAL VARIANCE G-4

APPENDIX H: WEIBULL GRAPH PAPER H-1

APPENDIX I: MEDIAN RANKS I-1

APPENDIX J – MIXTURES OF POPULATIONS AND FAILURE MODES J-1

- J.1 COMPETING RISK: J-1
- J.3 COMPETING RISK MIXTURE: J-2
- J. 4 COMPOUND COMPETING RISK MIXTURE: J-2
- J.5 WEIBATH MODEL: J-2
- J.7 CURVE SHAPE. J-3

APPENDIX K: ANSWERS TO PROBLEMS K-1

APPENDIX L: THE C4 FACTOR L-1

APPENDIX M: GRAPHICAL REPAIR ANALYSIS M-1

APPENDIX N: WALODDI WEIBULL N-1

REFERENCES R-1

INDEX I-1